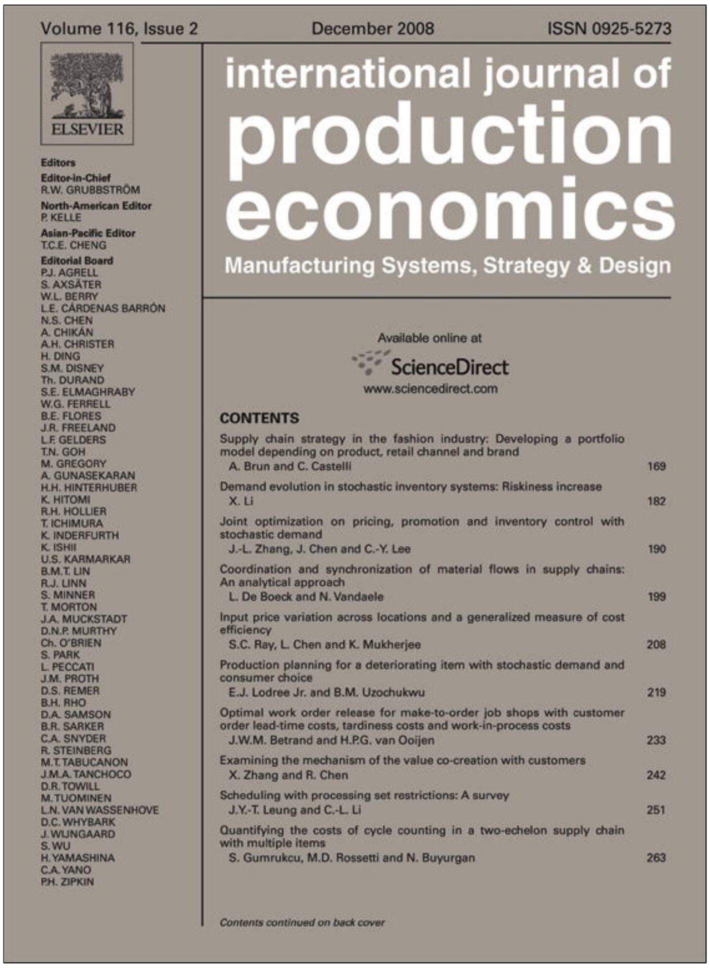

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Coordination and synchronization of material flows in supply chains: An analytical approach

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ABSTRACT

The coordination of joint material flows is a key element in supply chain management. Although analytical models for the coordination of materials are of great practical value, literature analyzing them remains scarce. This article contributes to this gap by studying a generic supply chain model. The supply chain is assumed to have a single production facility that is supplied by two independent suppliers. The field of combinatorics serves as a means to derive exact results for important performance measures, and the results suggest insights related to several supply chain management principles.

1. Introduction

Since the end of the 1990s, supply chain management has emerged as one of the key competitive elements for manufacturing and service companies. Stadtler and Kilger (2000) state that this enhanced competitiveness relies on two aspects: a closer integration of the organizations involved and the demand for better coordination of material, information, and financial flows.

In this article, we focus on the coordination of material flows. More specifically, we consider the necessity of materials synchronization and coordination in an assembly setting with two suppliers and one production facility in the supply chain. We assume the assembly setting has infinite warehousing capacity, and independent component arrival processes. Note that the case of independent component arrival processes is valid in mid- to long-term analysis of assembly settings as it points to the long run financial and physical limitations of the supply chain. Even in the short term, we believe that the independence assumption is realistic. That is, though components may be synchronized initially, when they require processing at the production facility in a variable environment before assembly takes place, they arrive virtually independent at the assembly process. In addition, companies are leaning more and more toward outsourcing many of their activities to other companies, sometimes in remote geographical locations. This makes the infinite warehousing assumption practical, because by sourcing globally, companies must maintain a global view on their supply chain operations to cope with the resultant uncertainties. Therefore, we are convinced that modeling these type of assembly settings is both appropriate and relevant.

By analyzing the dynamic effects of the coordination of material flows, we reveal one basic pitfall (supply flows must be synchronized in some way) and provide solutions to avoid it. Furthermore, we argue that in this context, the managerial decision maker relies on two generally accepted principles:

- Given the same supply frequency, companies prefer more reliable supply. Note that in general, each manufacturing environment prefers more reliable (i.e.

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Nomenclature

d_{cji} the interarrival time of component c (time between the ith and $i-1$th arrival of component c) in forming the jth pair of the joint probability distribution of the component arrival times for kit i

t'_{cji} the arrival time of component c for kit $i-1$ in forming the jth pair of the joint probability distribution of the component arrival times for kit i

t_{cji} the arrival time of component c of the jth pair of the joint probability distribution of the component arrival times for kit i

p_{ji} the probability of the jth pair of the joint probability distribution of the component arrival times for kit i

$E(W_{iv})_{\mathrm{U}}$ the expected value of the synchronization time of the components for kit i in case of discrete, uniformly distributed component interarrival times with v values

$\overline{E(W_{iv})}_{\mathrm{U}}$ the average of the expected values of the synchronization time of the components for the first i kits in case of discrete, uniformly distributed component interarrival times with v values

$E(T_{iv})_{\mathrm{U}}$ the expected value of the interarrival time of kit i at the assembly facility in case of discrete, uniformly distributed component interarrival times with v values

$\overline{E(T_{iv})}_{\mathrm{U}}$ the average of the expected values of the interarrival time for the first i kits at the assembly facility in case of discrete, uniformly distributed component interarrival times with v values

$E(W_{iv})_{\mathrm{G}}$ the expected value of the synchronization time of the components for kit i in case of discrete, generally distributed component interarrival times with v values

$\overline{E(T_{iv})}_{\mathrm{G}}$ the average of the expected values of the interarrival time for the first i kits at the assembly facility in case of discrete, generally distributed component interarrival times with v values

w_{ji} the synchronization time of the components for kit i corresponding to the jth pair in the joint probability distribution of the component arrival times for kit i

t_{ji} the arrival time of kit i corresponding to the jth pair in the joint probability distribution of the component arrival times for kit i

p_{ji} the probability of the jth pair in the joint probability distribution of the component arrival times for kit i

w_{si} the synchronization time of the components for kit i in state s of the probability distribution of the synchronization time of the components for kit i

t_{si} the arrival time of kit i in state s of the probability distribution of the arrival time of kit i

p_{si} the probability of the synchronization time of the components for kit i (arrival time of kit i) in state s of the probability distribution of the synchronization time of the components for kit i (the probability distribution of the arrival time of kit i)

$f(i,j)_1^{\mathrm{U}}$ the coefficient in row i and column j of the binomial triangle (discrete, uniform distribution with 2 values)

$f(i,j)_{v-1}^{\mathrm{U}}$ the coefficient in row i and column j of the v-nomial triangle (discrete, uniform distribution with v values)

$f(i,j)_{v-1}^{\mathrm{G}}$ the coefficient in row i and column j of the v-nomial triangle (discrete, general distribution with v values)

less variable) supply because increasing variability always degrades performance (see e.g. Hopp and Spearman, 2000).

- From time to time, the flow must be shut down (supply cap) (see e.g. Bonomi, 1987; Simon and Hopp, 1991) for both synchronized and unsynchronized supply, but determining the size of this supply cap is hard. An early shutdown could starve the system; a late shutdown may induce unnecessary inventory.

Next to these generally accepted principles, our study also reveals that more frequent supply (with the same reliability) does not lead to less synchronization stock.

In what follows, we derive basic results that underline these managerial principles from a generic assembly configuration.

We also note a two-fold additional motivation for studying this problem. First, we want to address the practical applicability of models in the coordination of materials, which is a key part of many supply chains. Models apply not only to typical production systems but also to services that 'assemble' the customer together with some resource or material, such as a passenger assembled with a transportation device. We think of a taxi driver waiting for passengers or passengers waiting for a taxi. Second, literature on the (exact) modeling of assembly settings is very scarce. Compared with other queueing topics which have been thoroughly studied in the literature, assembly settings have received much less attention. Moreover, the existing results mainly impose heavy assumptions on the system.

For ease of terminology, we refer in what follows to assembly systems, though we actually study supply chains for which coordination of materials is a major issue. We then organize our article as follows. In Section 2, we provide a detailed description of the assembly chain, its assumptions, related literature, and the notation we use. Section 3 provides explicit calculations for arriving at analytic results for the performance measures of the assembly system. In Sections 4 and 5, we discuss, respectively some special cases and important managerial insights. In Section 6, we conclude with our main results.

2. Definitions, assumptions, literature, and notations

We define assembly systems as systems in which the materials of two or more suppliers are joined together at the production facility. In this context, we define the materials of different suppliers as the components and the arrival process of the components from the different suppliers as the input processes. The process of joining the components together at the production facility is defined as the assembly process and the machine that performs this process is the assembly facility. The set containing all components designated for assembly is a kit, and an assembled kit refers to the resulting product of an assembly process. Such a system clearly requires that all components be simultaneously present which induces waiting time of the components for each other. We define this waiting time as the synchronization time. Assembly systems also entail an arrival process of kits at the assembly facility, in addition to the arrivals of the components at the production facility.

To gain insights into these two performance measures (i.e., the synchronization time of the components and the interarrival time of the kits), we limit our investigation to a generic assembly system (or generic supply chain system). This system is a basic version of the general assembly system, an open system with infinite warehouse capacity in which the components and kits are processed according to a first-come, first-served approach. The system contains two identical and independent supplier input processes, and each input process is subject to a renewal process. The component interarrival times for both suppliers are assumed to be discrete, uniformly distributed. We make this assumption for the following reasons.

- In practice, interarrival times are nearly always bounded between a minimum and a maximum value.
- The uniform distribution is a practical distribution with discrete and continuous versions.
- The analysis can be extended to general discrete distributions (see Section 4).
- The logic behind the analysis also applies to general, continuous distributions (De Boeck, 2003).

We assume equal distances between the discrete values of the uniform distribution (see also Section 4). Furthermore, we presume that a kit consists of exactly one component that comes from each input process. We hypothetically split the warehouse (assembly queue) into two separate warehouses, as we illustrate in Fig. 1, to clarify the detailed process of assembling both components.

The synchronization warehouse contains the components that wait for forming a kit; the assembly warehouse contains the kits that wait for their operation at the assembly facility.

We subsequently develop formulas for the synchronization time of the components and the interarrival time of the kits. Once we know these times, we can derive the performance of the entire system by applying the open queueing network model described by Vandaele (1996), Lambrecht et al. (1998), and Vandaele et al. (2002). This model already has been proven to provide good results for systems without assemblies (Vandaele et al., 1999, 2000).

Relevant literature on similar assembly systems starts with a paper by Harrison (1973). In his pioneering work, he states that an assembly system with k renewal input processes and a single assembly facility requiring one component of each class is unstable when there is no restriction on system capacity. The paper of Crane (1974) is a multi-facility generalization of the paper by Harrison (1973). Latouche (1981) obtains the distribution of the number of products in the system assuming two Poisson input processes and exponential processing times under different arrival mechanisms. Bhat (1986) provides the distribution of the lead time as well as the stationary probability vector of the queue length and controls the input processes by putting a limit on the number of components in each class. Lipper and Sengupta (1986) as well as Bonomi (1987) offer, respectively approximations, a procedure for computing the throughput and mean inventory by controlling the input in some way. Hopp and Simon (1989, 1993), Simon and Hopp (1991, 1995), Duenyas and Hopp (1992, 1993), and Duenyas and Keblis (1995) compute the throughput and/or inventory of

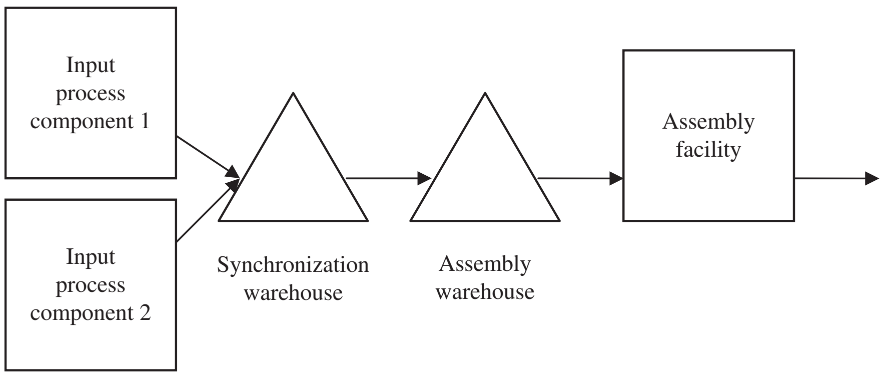

Fig. 1. Generic assembly system with two input processes and a synchronization and an assembly warehouse.

assembly systems for different component input processes subject to different mechanisms of input control. Baker et al. (1990, 1993) and Baker and Powell (1995) use simulation to study different aspects of assembly systems such as the effect of variability on the efficiency of assembly lines, the allocation of work in order to maximize throughput and throughput of unbalanced three-station assembly systems. Som et al. (1994) characterize the arrival process of kits at the assembly facility assuming exponential component interarrival times. Powell and Pyke (1998) indicate how optimally deploying limited buffers capacity. Wilhelm and Som (1999) approximate the probability of kit inventory and end-product inventory for generally and independently distributed assembly times. Takahashi et al. (2000) model the kit arrival process at the assembly facility assuming a Poisson and PH-renewal input process and finite buffer capacities. Sabuncuoglu et al. (2002) study the effect of the number of component input processes, work transfer, processing time distributions, buffers, and buffer allocation schemes on throughput and interdeparture time variability of assembly systems. Ramachandran and Delen (2005) investigate the dynamics involved in an assembly system with two independent input processes having state-dependent arrival intensities.

The above literature reveals that modeling assembly systems in an exact analytic way is extremely difficult, mainly because of the system state space, the mathematical complexity, and the instability of assembly systems when warehouses have an infinite capacity and the input processes are independent renewal processes (e.g. Bhat, 1986; Simon and Hopp, 1995). Harrison (1973) even proves that in the latter case, synchronization times do not converge unless the buffers are finite. Therefore, if infinite buffers are assumed, a kind of input control always appears in the preceding literature to preserve system stability. We achieve input control by analyzing an assembly system with finite time horizons (e.g. days, weeks); that is, we assume a finite number of kits.

In view of these assumptions, we can formulate our precise goal. We attempt to find exact results for the synchronization time of the components and the interarrival time of the kits at the assembly facility as functions of (1) the index of the kit (because we want to get an idea of the results for a limited number of kits) and (2) the number of discrete values of the discrete, uniformly distributed component interarrival times (which will offer insight into continuous, uniformly distributed component interarrival times).

3. Exact analysis of the synchronization time of components and the interarrival time of kits at the assembly facility

We base our subsequent analysis on the generic assembly system described in Section 2.

3.1. Preliminary analysis

To make the calculations comprehensive, we illustrate our analysis with a simple example. We assume discrete, uniformly distributed component interarrival times with two values ($v = 2$), namely, 8 and 9 time units.

When both components start to be supplied at time 0, there are four (v^2) pairs in the joint probability distribution of the component arrival times for the first kit, as we illustrate in Table 1. Note that a pair consists of one value for the arrival time of component 1 and one value for the arrival time of component 2 (in forming kit 1).

When we generate the second kit, there are 16 (v^4) pairs in the joint probability distribution of the component arrival times for the second kit: each of the 4 pairs in the joint probability distribution of the component arrival times for the first kit can be combined with 4 pairs of interarrival times for both components for the second kit ((8,8), (8,9), (9,8) and (9,9)). The interarrival time of component 1 (2) for the second kit equals the time between the first and the second arrival of component 1 (2). By summing the arrival time of component 1 (2) for the first kit and the interarrival time of component 1 (2) for the second kit, we obtain the arrival time for component 1 (2) in forming the second kit. This is represented in Table 2.

Remark that not all pairs are different. As such, the pair (16,17) appears twice in the joint probability distribution of the component arrival times for the second kit.

In general, we have v^{2i} pairs in the joint probability distribution of the component arrival times for kit i.

The formula for the expected value of the synchronization time of the components for kit i then becomes

$$E(W_{iv})_{\mathrm{U}} = \sum_{j=1}^{(v^2)^i} w_{ji} p_{ji} \text{ with } w_{ji} = |t_{1ji} - t_{2ji}| \tag{1}$$

which is the sum of all synchronization times (w_{ji}) (each synchronization time w_{ji} corresponding to the absolute value of the difference of both component arrival times of pair j in the joint probability distribution of the component arrival times for kit i) multiplied by their respective probabilities p_{ji} (which is the probability of the jth pair in the joint probability distribution of the component arrival times for kit i).

If we take an average value over the expected values of the synchronization time of the components for kit $1, 2, \ldots, i$, we get

$$\overline{E(W_{iv})_{\mathrm{U}}} = \frac{1}{i} \sum_{k=1}^{i} E(W_{kv})_{\mathrm{U}}. \tag{2}$$

For the average value of the expected values of the interarrival times of the first i kits at the assembly facility,

Table 1
Joint probability distribution of the component arrival times for the first kit for discrete, uniformly distributed component interarrival times with values 8 and 9

j	t_{1j1}	t_{2j1}	p_{j1}
1	8	8	0.25
2	8	9	0.25
3	9	8	0.25
4	9	9	0.25

Table 2
Joint probability distribution of the component arrival times for the second kit for discrete, uniformly distributed component interarrival times with values 8 and 9

j	t'_{1j1}	t'_{2j1}	d_{1j2}	d_{2j2}	p_{j2}	t_{1j2}	t_{2j2}
1	8	8	8	8	0.0625	16	16
2	8	8	8	9	0.0625	16	17
3	8	8	9	8	0.0625	17	16
4	8	8	9	9	0.0625	17	17
5	8	9	8	8	0.0625	16	17
6	8	9	8	9	0.0625	16	18
7	8	9	9	8	0.0625	17	17
8	8	9	9	9	0.0625	17	18
9	9	8	8	8	0.0625	17	16
10	9	8	8	9	0.0625	17	17
11	9	8	9	8	0.0625	18	16
12	9	8	9	9	0.0625	18	17
13	9	9	8	8	0.0625	17	17
14	9	9	8	9	0.0625	17	18
15	9	9	9	8	0.0625	18	17
16	9	9	9	9	0.0625	18	18

we have that

$$\overline{E(T_{iv})}_{\mathrm{U}} = \frac{1}{i}\sum_{j=1}^{(v^2)^i} t_{ji}p_{ji} \text{ with } t_{ji} = \max(t_{1ji}, t_{2ji}) \tag{3}$$

which is $1/i$ multiplied by the sum of all arrival times (t_{ji}) (each arrival time t_{ji} corresponding to the maximum of both component arrival times of pair j in the joint probability distribution of the component arrival times for kit i) multiplied by their respective probabilities p_{ji} (which is the probability of the jth pair in the joint probability distribution of the component arrival times for kit i).

Since we know that that $\overline{E(T_{iv})}_{\mathrm{U}} = (1/i)\sum_{k=1}^{i} E(T_{kv})_{\mathrm{U}}$ we can easily obtain a formula for the expected value of the interarrival time of kit i at the assembly facility

$$E(T_{iv})_{\mathrm{U}} = i\overline{E(T_{iv})}_{\mathrm{U}} - (i-1)\overline{E(T_{(i-1)v})}_{\mathrm{U}}. \tag{4}$$

Note that $E(X_{iv})_{\mathrm{U}}$ represents the expected value of the variable X for a specific kit i whereas $\overline{E(X_{iv})}_{\mathrm{U}}$ represents the average of all expected values $E(X_{kv})_{\mathrm{U}}$ ($k = 1\ldots i$) for the first i kits assuming that each expected value has a probability of $1/i$.

In the following discussion, we concentrate on Eqs. (1) and (3) since they can be derived easily from the joint probability distribution of the component arrival times for kit i. Note that Eqs. (2) and (4) can be obtained easily from Eqs. (1) and (3). If we apply Eqs. (1) and (3) to the first kit of our example, we arrive at the results in Table 3.

Following the same reasoning, we can obtain results for the performance measures of the subsequent kits ($i>1$). In general, we can perform this analysis for all discrete, uniformly distributed component interarrival times with v values. Nevertheless, as i and v rise, the number of pairs in the joint probability distribution of the component arrival times for kit i becomes progressively larger, which eventually makes it impossible to continue the analysis. Moreover, a mathematical structure within the pairs prevents us from enumerating all pairs, as we will discuss next in Section 3.2.

Table 3
Expected value of the synchronization time of the components for the first kit and the average of the expected values of the interarrival time of one kit in case of discrete, uniformly distributed component interarrival times with values 8 and 9

j	w_{j1}	t_{j1}	p_{j1}	$E(W_{12})_{\mathrm{U}}$	$\overline{E(T_{12})}_{\mathrm{U}}$
1	0	8	0.25		
2	1	9	0.25		
3	1	9	0.25		
4	0	9	0.25	0.5	8.75

3.2. Combinatorial analysis

The mathematical structure within the pairs of the joint probability distribution of the component arrival times for kit i is determined by the multinomial triangles. To illustrate this structure, we again rely on our previous example. The structure in this example is based completely on Pascal's triangle (i.e., the binomial triangle), and if we omit the first row of this triangle, we obtain (in numbers and binomial coefficients)

$$\begin{array}{ccc} 1\ 1 & \text{or} & C_0^1C_1^1 \\ 1\ 2\ 1 & & C_0^2C_1^2C_2^2 \\ 1\ 3\ 3\ 1\ldots & & C_0^3C_1^3C_2^3C_3^3\ldots \end{array}$$

The link between the rows of this triangle and the pairs in the joint probability distribution of the component arrival times for kit i should be elaborated for the second kit (because the link cannot be perceived easily for the first kit). To illustrate the structure for the second kit, we first reorganize Table 2 by ordering the different arrival times of the first component in increasing order. If these arrival times are the same, we order the arrival times of the second component in increasing order, as we show in Table 4.

The symbol # in Table 4 refers to the number of occurrences of all different pairs. Since from this point on, we are only interested in all different pairs, we will introduce the name 'state'. A state in a joint probability distribution is defined as an ordered pair. Here, a state in the joint probability distribution of the component arrival times for kit i is an ordered pair consisting of an arrival time of component 1 and an arrival time of component 2. The set of states in this joint probability distribution then equals the set of all different pairs. (A state in a general probability distribution is defined as one element of this distribution.) To illustrate the link, we offer two additional definitions. We define a block as the set of states with the same value for the arrival time of the first component for a kit and an element as a state within a specific block for a kit. Thus, we distinguish three blocks in the states of the joint probability distribution of the component arrival times for the second kit, each block having three elements. As a brief example, we note that (t_{1s2},t_{2s2},) equal to (16,16) is the state corresponding to the first element of the first block. Because we are working with the second kit, we take the second row in Pascal's triangle, or 1 2 1, and write this row as a column vector multiplied by the same row

Table 4
Pattern in the different states of the joint probability distribution of the component arrival times for the second kit in case of discrete, uniformly distributed component interarrival times with values 8 and 9

s	Block	Element	t_{1s2}	t_{2s2}	#
1	1	1	16	16	1
2	1	2	16	17	2
3	1	3	16	18	1
4	2	1	17	16	2
5	2	2	17	17	4
6	2	3	17	18	2
7	3	1	18	16	1
8	3	2	18	17	2
9	3	3	18	18	1

as a row vector, which gives us

$$\begin{bmatrix}\mathbf{1}\\\mathbf{2}\\\mathbf{1}\end{bmatrix}\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{1}\end{bmatrix}=\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{1}\\\mathbf{2} & \mathbf{4} & \mathbf{2}\\\mathbf{1} & \mathbf{2} & \mathbf{1}\end{bmatrix}$$

or

$$\begin{bmatrix}\mathbf{C_0^2}\\\mathbf{C_1^2}\\\mathbf{C_2^2}\end{bmatrix}\begin{bmatrix}\mathbf{C_0^2} & \mathbf{C_1^2} & \mathbf{C_2^2}\end{bmatrix}=\begin{bmatrix}\mathbf{C_0^2C_0^2} & \mathbf{C_0^2C_1^2} & \mathbf{C_0^2C_2^2}\\\mathbf{C_1^2C_0^2} & \mathbf{C_1^2C_1^2} & \mathbf{C_1^2C_2^2}\\\mathbf{C_2^2C_0^2} & \mathbf{C_2^2C_1^2} & \mathbf{C_2^2C_2^2}\end{bmatrix}.$$

Note that the rows (columns) of the resulting matrices correspond exactly to the values in column # in Table 4. That is, the number of times the states corresponding to each element in each block appear, can be represented as a multiplication of two binomial coefficients.

If we define the kits by index i, the elements by index e, and the blocks by index b, we can express in general the number of times the state corresponding to element e of block b appears in the joint probability distribution of the component arrival times for kit i as $C^i_{b-1}C^i_{e-1}$ and the probability of that state as $(C^i_{b-1}C^i_{e-1})/(v^2)^i$.

We thus can prove that for discrete, uniformly distributed component interarrival times with two values, the preceding expression is always valid by ordering these arrival times, as we explained previously. The only parameter that changes with index i is the arrival times of both components. Therefore, we obtain the following lemma.

Lemma 1. *For discrete, uniformly distributed component interarrival times with two values, we have that the number of times that the state corresponding to element e in block b appears in the joint probability distribution of the component arrival times for kit i equals* $C^i_{b-1}C^i_{e-1}$.

De Boeck (2008) offers a proof by induction.

Furthermore, we define the v-nomial coefficient $f(i,j)^{\mathrm{U}}_{v-1}$ as the coefficient in row i and column j of the v-nomial triangle. Note that the 2-nomial (or binomial) coefficients can then be represented as $f(i,j)^{\mathrm{U}}_1$. Along similar lines, we obtain another lemma.

Lemma 2. *For discrete, uniformly distributed component interarrival times with v values, we have that the number of times that the state corresponding to element e in block b appears in the joint probability distribution of the component arrival times for kit i equals* $f(i,b-1)^{\mathrm{U}}_{v-1}\,f(i,e-1)^{\mathrm{U}}_{v-1}$.

Again, De Boeck (2008) offers a proof by induction for this lemma.

If we use these multiplications of the v-nomial coefficients to express the probability of each state, we find general expressions for $E(W_{iv})_{\mathrm{U}}$ and $\overline{E(T_{iv})}_{\mathrm{U}}$

$$E(W_{iv})_{\mathrm{U}} = \sum_{s=1}^{i(v-1)+1} w_{si}p_{si} \tag{5}$$

which is the sum of all different synchronization times (w_{si}) multiplied by their respective probabilities p_{si} (which is the probability of state s in the synchronization time probability distribution of the components for kit i) and

$$\overline{E(T_{iv})}_{\mathrm{U}} = \sum_{s=1}^{i(v-1)+1} \frac{t_{si}}{i}p_{si} \tag{6}$$

which is the sum of all different interarrival times (t_{si}/i) multiplied by their respective probabilities p_{si} (which is the probability of state s in the interarrival time probability distribution of kit i). We note here that p_{si} is different in formulas (5) and (6)

In turn, we can apply Eqs. (5) and (6) to our example for the second kit, as we represent in Table 5, which we base on Table 4.

If we structure $E(W_{22})_{\mathrm{U}}$ as in Eq. (5), we achieve

$$E(W_{22})_{\mathrm{U}} = \sum_{q=1}^{2} q\,\frac{\sum_{j=q}^{2} 2f(2,j)^{\mathrm{U}}_1 f(2,j-q)^{\mathrm{U}}_1}{(2^2)^2}. \tag{7}$$

Note that we omit a synchronization time of zero, since it would cancel out in the summation. If we conduct the same analysis for $\overline{E(T_{22})}_{\mathrm{U}}$ using Eq. (6), we find

$$\overline{E(T_{22})}_{\mathrm{U}} = \sum_{q=0}^{2} \frac{q+16}{2}\,\frac{\sum_{j=0}^{q} 2f(2,j)^{\mathrm{U}}_1 f(2,q)^{\mathrm{U}}_1 - f(2,q)^{\mathrm{U}}_1}{(2^2)^2}. \tag{8}$$

In general, we can prove the following proposition for the expected value of the synchronization time of the components for each kit i.

Table 5
The combinatorial pattern in the different states of the joint probability distribution of the component arrival times for the second kit in case of discrete, uniformly distributed component interarrival times with values 8 and 9 and the related synchronization times of the components for the second kit and the interarrival times for the first two kits at the assembly facility

s	t_{1s2}	t_{2s2}	#	w_{s2}	t_{s2}
1	16	16	$f(2,0)^{\mathrm{U}}_1 f(2,0)^{\mathrm{U}}_1$	0	16
2	16	17	$f(2,0)^{\mathrm{U}}_1 f(2,1)^{\mathrm{U}}_1$	1	17
3	16	18	$f(2,0)^{\mathrm{U}}_1 f(2,2)^{\mathrm{U}}_1$	2	18
4	17	16	$f(2,1)^{\mathrm{U}}_1 f(2,0)^{\mathrm{U}}_1$	1	17
5	17	17	$f(2,1)^{\mathrm{U}}_1 f(2,1)^{\mathrm{U}}_1$	0	17
6	17	18	$f(2,1)^{\mathrm{U}}_1 f(2,2)^{\mathrm{U}}_1$	1	18
7	18	16	$f(2,2)^{\mathrm{U}}_1 f(2,0)^{\mathrm{U}}_1$	2	18
8	18	17	$f(2,2)^{\mathrm{U}}_1 f(2,1)^{\mathrm{U}}_1$	1	18
9	18	18	$f(2,2)^{\mathrm{U}}_1 f(2,2)^{\mathrm{U}}_1$	0	18

Proposition 1.

$$E(W_{iv})_{\mathrm{U}} = \sum_{q=1}^{(v-1)i} \frac{q(\mathrm{Ub}-\mathrm{Lb})}{v-1} \frac{\sum_{j=q}^{(v-1)i} 2f(i,j)_{v-1}^{\mathrm{U}} f(i,j-q)_{v-1}^{\mathrm{U}}}{v^{2i}}, \tag{9}$$

where Ub *is the upper bound and* Lb *the lower bound of the uniform distribution.*

We express the expected value of the synchronization time in terms of i and v. In addition, the first part of this exact result—$q(\mathrm{Ub}–\mathrm{Lb})/(v–1)$—represents the different synchronization times in the synchronization time probability distribution of the components for kit i, whereas the second part depicts their respective probabilities.

We can also prove the following proposition for the average of the expected values of the interarrival time of the first i kits at the assembly facility.

Proposition 2.

$$\overline{E(T_{iv})_{\mathrm{U}}} = \sum_{q=0}^{(v-1)i} \frac{q(\mathrm{Ub}-\mathrm{Lb})/(v-1) + i\mathrm{Lb}}{i} \frac{\sum_{j=0}^{q} 2f(i,j)_{v-1}^{\mathrm{U}} f(i,q)_{v-1}^{\mathrm{U}} - [f(i,q)_{v-1}^{\mathrm{U}}]^2}{v^{2i}}. \tag{10}$$

Proofs of Eqs. (9) and (10) appear in De Boeck (2008).

4. Special cases

For the analysis in Section 4, we assume equal distances between the discrete values of the uniform distribution (see Section 2). However, within these distances, we may experience the following variants.

- In case of discrete, uniformly distributed component interarrival times for which the difference between two successive values equals one time unit (i.e., units equidistant), Eqs. (9) and (10) can be simplified, such that $q(\mathrm{Ub}–\mathrm{Lb})/(v–1)$ becomes q, because $v–1$ and Ub–Lb vanish in this case.
- In case of discrete, uniformly distributed component interarrival times for which the difference between two successive values is unequal, Eqs. (9) and (10) do not apply, because of the number of states, the number of different synchronization times, and the different arrival times for a specific value of i and v ($v>2$). This number of states is always larger when the differences between two successive values of the discrete, uniform distribution are unequal. Therefore, the general structure that relates to the number of times the states occur in the joint probability distribution of the component arrival times for a specific kit does not apply.

Continuous, uniformly distributed component interarrival times are defined on intervals that contain an infinite number of values. Therefore, by taking the limit for $v\to\infty$ in Eqs. (9) and (10), we might achieve results for the continuous case. In this case, no results for the multinomial coefficients can be obtained. Nevertheless, we may derive results for the continuous case by writing the performance measures as a function of v for a specific kit i. In turn, we obtain results for our example (discrete, uniform distribution with Lb equal to 8 and Ub equal to 9 for the component interarrival times) by expressing $E(W_{iv})_{\mathrm{U}}$ and $\overline{E(T_{iv})_{\mathrm{U}}}$ (for $i=1, 2$) as a function of v. The results are as follows:

$$E(W_{1v})_{\mathrm{U}} = \frac{1}{3}\left[\frac{v+1}{v}\right], \tag{11}$$

$$E(W_{2v})_{\mathrm{U}} = \frac{1}{15}\left[\frac{7(v-1)^3+28(v-1)^2+37(v-1)+18}{v^3}\right], \tag{12}$$

$$\overline{E(T_{1v})_{\mathrm{U}}} = \frac{1}{6}\left[\frac{52(v-1)+53}{v}\right], \tag{13}$$

$$\overline{E(T_{2v})_{\mathrm{U}}} = \frac{1}{60}\left[\frac{517(v-1)^3+1558(v-1)^2+1567(v-1)+528}{v^3}\right]. \tag{14}$$

When we take the limit for $v\to\infty$ in Eqs. (11)–(14), we emerge with the results for the continuous case. These results are crucial in assessing the approximations of both performance measures when the component interarrival times are uniform, continuous distributions.

For discrete, uniformly distributed component interarrival times with two values, Eqs. (9), (2), (4), and (10) can be simplified, respectively, as follows:

$$E(W_{i2})_{\mathrm{U}} = (\mathrm{Ub}-\mathrm{Lb})\frac{\Gamma(i+0.5)}{\pi^{1/2}\Gamma(i)}, \tag{15}$$

$$\overline{E(W_{i2})_{\mathrm{U}}} = (\mathrm{Ub}-\mathrm{Lb})\frac{(2i+1)\Gamma(i+0.5)}{3i\pi^{1/2}\Gamma(i)}, \tag{16}$$

$$E(T_{i2})_{\mathrm{U}} = \frac{\mathrm{Lb}+\mathrm{Ub}}{2} + 0.5(\mathrm{Ub}-\mathrm{Lb})\frac{i\Gamma(i+0.5)}{(2i-1)\pi^{1/2}\Gamma(i+1)}, \tag{17}$$

and

$$\overline{E(T_{i2})_{\mathrm{U}}} = \frac{\mathrm{Lb}+\mathrm{Ub}}{2} + 0.5(\mathrm{Ub}-\mathrm{Lb})\frac{\Gamma(i+0.5)}{\pi^{1/2}\Gamma(i+1)}, \tag{18}$$

where $\Gamma(x)$ is the gamma function of x.

If we plot the results of these equations for the first 20 kits of our example, we obtain the graphs in Fig. 2.

As we clearly observe in Fig. 2, the synchronization time diverges (in line with Harrison, 1973), whereas the interarrival time converges. We also can confirm this behavior by taking the limit of Eqs. (15)–(18) for $i\to\infty$, which leads both $E(W_{i2})_{\mathrm{U}}$ and $\overline{E(W_{i2})_{\mathrm{U}}}$ to infinity, while the asymptotic value (Lb+Ub)/2 applies to both $E(T_{i2})_{\mathrm{U}}$ and $\overline{E(T_{i2})_{\mathrm{U}}}$.

The analysis pertaining to the discrete, generally distributed case falls in line with the analysis developed in Section 3.2. Remark that for the discrete, uniform case, the coefficients of $x^j (j=0,1,\ldots,(v-1)i)$ of the polynomial $(x^0+x^1+x^2+x^3+\cdots+x^{v-1})^i$ constitute row i in the v-nomial triangle. We represent these coefficients by $f(i,j)_{v-1}^{\mathrm{U}}$. The coefficients in this polynomial are 1, because all values in the discrete, uniform distribution have a probability of $1/v$. In the discrete, general case with v values, we express the probabilities of the kth value as $a_k/(a_1+a_2+\cdots+a_k+\cdots+a_v)$. Then, the coefficients we need to find the probabilities

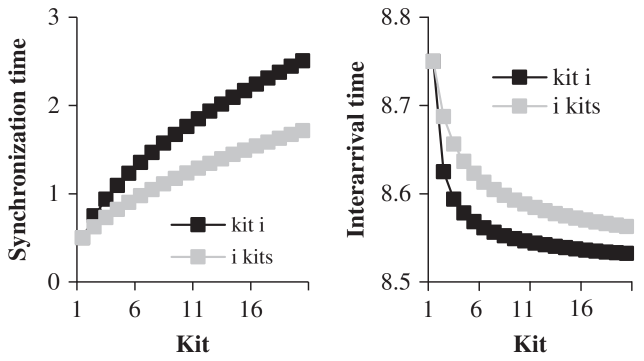

Fig. 2. Expected value (represented by 'kit i') and the average of the expected values (represented by 'i kits') of the synchronization time of the components and the interarrival time for the first 20 kits in case of discrete, uniformly distributed component interarrival times with values 8 and 9.

of the different states in the joint probability distribution of the components for kit i correspond to the coefficients of x^j $(j=0,1,\ldots,(v-1)i)$ of the polynomial $(a_1x^0+a_2x^1+a_3x^2+a_4x^3+\cdots+a_vx^{v-1})^i$. If we define these coefficients as $f(i,j)^G_{v-1}$, we can derive the following equations, analogous to Eqs. (9) and (10), for the discrete, general case:

$$E(W_{iv})_G = \sum_{q=1}^{(v-1)i} \frac{q(Ub-Lb)}{v-1} \frac{\sum_{j=q}^{(v-1)i} 2f(i,j)^G_{v-1} f(i,j-q)^G_{v-1}}{(a_1+a_2+a_3+\cdots+a_v)^{2i}}, \tag{19}$$

and

$$\overline{E(T_{iv})}_G = \sum_{q=0}^{(v-1)i} \frac{q(\mathrm{Ub}-\mathrm{Lb})/(v-1)+iLb}{i} \times \frac{\sum_{j=0}^{q} 2f(i,j)^G_{v-1} f(i,j-q)^G_{v-1} - [f(i,q)^G_{v-1}]^2}{(a_1+a_2+a_3+\cdots+a_v)^{2i}}. \tag{20}$$

5. Important managerial insights

The above formulas allow deriving some managerial issues we listed in the introduction.

First, given the same supply frequency, companies will prefer more reliable supplies. More reliable supplies mean that the range of the component interarrival times (Ub–Lb) becomes smaller; more frequent supplies mean that the average component interarrival time ((Ub+Lb)/2) decreases. Moreover, a more reliable supply leads to a slower divergence of the synchronization time (less synchronization stock) and a faster convergence of the kit interarrival times (fewer outages due to material shortages). Therefore, more reliable supply translates into less synchronization stock and improved productivity. At the limit, if we were to remove unreliability, the synchronization time becomes zero, and the interarrival time immediately achieves its lower boundary. This scenario would mean no synchronization stock and no productivity losses.

Second, more frequent supply with the same reliability has no influence on the values or speed of divergence of the synchronization time. Although it affects the values of the kit interarrival times, it does not influence the speed of their convergence. More frequent supply therefore leads to lower values of the kit interarrival times.

Third, if supply is not synchronized somehow, the synchronization time grows to infinity. Therefore, a flow control measure is needed. In the existing literature, we can mainly distinguish 4 types of input control mechanisms. A first one is a limit on the buffer capacity in combination with a removal of components from the input processes (see e.g. Lipper and Sengupta, 1986) or a shut down of the input processes (see e.g. Simon and Hopp, 1995) when the maximum buffer capacity is reached. A second mechanism is letting depend the component arrivals on (see e.g. Latouche, 1981) or shutting down the component input process when (see e.g. Bonomi, 1987) there is a specified excess of the number of one component in the system over the others. A third one is a limit on the number of components in the system (see e.g. Bhat, 1986). The fourth type uses control mechanisms as bins (see e.g. Hopp and Simon, 1989), conwip (see e.g. Duenyas and Hopp, 1993) or kanban (see e.g. Duenyas and Keblis, 1995). For our system, we propose a control mechanism related to the third type. Note that apart from type one (which assumes finite buffers) and type four (which induces production control mechanisms), the other two relate to a controlled supply and a shut down (shipments are stopped) if the synchronization stock reaches a particular cap, which enables the stock to be consumed and therefore supply to be resumed. This cap can be viewed as a finite number of arrived kits. The cap must be dimensioned, such that it reasonably avoids a too high synchronization stock but still provides sufficient protection of the assembly system (i.e., keeps the average kit interarrival time low). The decision about the size of the supply cap remains challenging, but it may be based on the insight depicted in Fig. 2. An early shutdown will starve the system, and a late shutdown likely will create unneeded inventory.

6. Conclusions

We analyze the coordination of material from two independent suppliers, which is then assembled in an assembly facility. In so doing, we find that the assumption of independent renewal interarrival times of the material from both suppliers to the assembly facility, combined with infinite warehouse capacity, leads to diverging values for the synchronization time of the components and converging values for the interarrival time of the kits at the assembly facility (to the average value of the uniform distribution).

From a managerial point of view, we derive the following implications:

- More reliable supply (with the same frequency) leads to less synchronization stock and improved productivity.
- More frequent supply (with the same reliability) does not lead to less synchronization stock.

- Supply must be synchronized and needs a shutdown cap. Early shutdowns cause the system to starve; late shutdowns induce unneeded inventory.

In every case, we show that a limit on input flows is mandatory, typically through some form of input control. Such a limit preserves the system with the diverging synchronization effect while counting on the converging interarrival effect. Both effects are adverse but influence the efficiency and productivity of the supply chain. The former points to the time lost in synchronization stock, and the latter is the goal of supply chain coordination.

The exact results provided herein represent a first step of a general analysis. The next step will use the logic behind the discrete case to derive satisfying approximations of the continuous, general case. Thus, in further research, we will focus on approximations, because exact analysis is impossible. The absence of exact relations also can be observed in the literature, which mainly reports bounds and approximations (see e.g. Hopp and Simon, 1989), whereas research studying more realistic situations limits its analysis to simulation (see e.g. Baker et al., 1990).

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